From the Type I String to M-Theory:
A Continuous Connection

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Abstract

It is well-known that the $SO(32)$ and $E_8 \times E_8$ heterotic strings can be continuously connected to each other in nine dimensions. Since the strong-coupling duals of these theories are respectively the $SO(32)$ Type I theory and M-theory compactified on a line segment, there should be a corresponding continuous connection between the Type I string and M-theory. In this paper, we give an explicit construction of this dual connecting theory. Our construction also enables us to realize the $E_8 \times E_8$ heterotic string as a D-string soliton of the Type I theory. This provides a useful alternative description of the D-brane bound states previously discussed from a Type I$'$ point of view.

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1 Introduction

In ten dimensions, there are only two supersymmetric heterotic string theories: the $SO(32)$ theory, and the $E_8 \times E_8$ theory. These theories have, however, very different strong-coupling duals: the $SO(32)$ theory is believed to be dual to the $SO(32)$ Type I theory [1, 2], while the $E_8 \times E_8$ theory is believed to behave at strong coupling as a certain eleven-dimensional theory (M-theory) compactified on a line segment [3]. This difference in the dual theories is particularly striking when one considers the fact that the $SO(32)$ and $E_8 \times E_8$ heterotic strings are closely related in nine dimensions. In fact, there exist two types of relations between these theories in nine dimensions. The first is a *discrete* T-duality relation: when each theory is compactified on a circle and is subjected to a Wilson line that breaks its gauge symmetry to $SO(16) \times SO(16)$, the resulting theories are T-dual to each other. Second, there is also a *continuous* relation between these two theories in nine dimensions: as first discussed in Refs. [4, 5], they can be continuously connected to each other through deformations of background fields. Both of these connections imply that nine-dimensional compactifications of M-theory should be closely related to those of the Type I string.

In the case of the T-duality relation, this expectation is indeed borne out by the following well-known observation. The $E_8 \times E_8$ string, when compactified on a circle, has a strong-coupling dual which can be identified as M-theory compactified first on a circle and then on a line segment. However, M-theory compactified on a circle yields the Type IIA string, and the Type IIA theory compactified on a line segment is equivalent to the T-dual of the Type I theory, also known as the Type I$'$ theory. Thus, since the Type I theory is the strong-coupling dual of the $SO(32)$ heterotic string, we see that the same T-duality relation that exists between the $SO(32)$ and $E_8 \times E_8$ heterotic strings in nine dimensions also exists for their strong-coupling duals.

Until now, however, the second relationship has not been demonstrated. Specifically, no *continuous* nine-dimensional connection has been given that relates the Type I string to an M-theory compactification. It is the purpose of this note to explicitly construct this continuous connection.

Our approach will be as follows. First, we shall explicitly construct a nine-dimensional heterotic string model which continuously interpolates between the ten-dimensional $SO(32)$ and $E_8 \times E_8$ heterotic strings. Our model will be parametrized by the radius R of the compactified dimension: as $R \to \infty$, our interpolating model will reproduce the ten-dimensional supersymmetric $SO(32)$ heterotic string, while as $R \to 0$, our interpolating model will reproduce the $E_8 \times E_8$ string. Note that this supersymmetric interpolating model is similar to the *non*-supersymmetric interpolating models that have recently been used [6, 7] for deriving the strong-coupling duals

of *non*-supersymmetric heterotic strings.

Given this heterotic interpolating model, we shall then proceed to construct its strong-coupling dual. Our methods will be similar to those employed in Refs. [6, 7]. Thus, our dual theory will in some sense interpolate between the Type I theory and M-theory, or more specifically between the Type I theory and the Type I$'$ theory at infinite coupling. This situation is illustrated in Fig. 1.
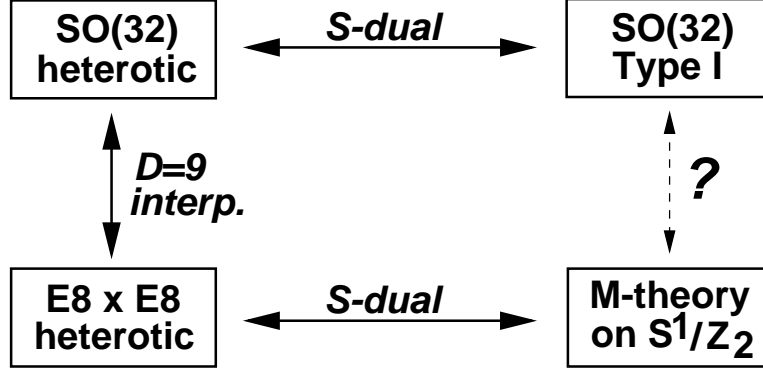
Figure 1: We construct a strong-coupling dual for the nine-dimensional heterotic string model that continuously interpolates between the ten-dimensional heterotic $SO(32)$ and $E_8 \times E_8$ string models. This dual theory thereby provides a continuous interpolation between the Type I string and M-theory compactified on a line segment. Note that the $E_8 \times E_8$ string is *equivalent* to M-theory compactified on a line segment. In this figure, by contrast, the lower right box refers to the theory produced by compactifying M-theory first on a circle, then on a line segment, and then taking the radius of the circle to infinity. This is equivalent to the Type I$'$ theory at infinite coupling.

Once we have constructed this dual theory, we will examine its D1-brane soliton. We will find that this soliton reproduces the worldsheet dynamics of the $E_8 \times E_8$ heterotic string theory. Note that because the $E_8 \times E_8$ theory compactified on a circle is related by strong/weak coupling duality to the Type I$'$ string, it is expected that there exist D-particle bound states in the Type I$'$ theory that give rise to the $E_8 \times E_8$ gauge symmetry. These states have been discussed in the context of the Type I$'$ theory in Ref. [8]. In this paper, by contrast, we will demonstrate that the D1-brane soliton of our Type I dual interpolating model gives rise to the expected states. Thus, our approach — which is based entirely on the Type I point of view — provides the desired continuous connection between the Type I string and M-theory.

2 Connecting $SO(32)$ to $E_8 \times E_8$: The Heterotic Interpolating Model

We begin by constructing our heterotic interpolating model that continuously connects the ten-dimensional supersymmetric $SO(32)$ heterotic string and the ten-dimensional supersymmetric $E_8 \times E_8$ heterotic string.

To do this, we start with the ten-dimensional $SO(32)$ heterotic theory, and we compactify this theory on a circle of radius R_H. Next, we orbifold the resulting nine-dimensional theory by the $\mathbb{Z}_2$ element $Q'_H \equiv \mathcal{T} Q_H$ defined as follows. The operator $\mathcal{T}$ is a translation of the coordinate x_1 of the circle by half of the circumference of the circle:

$$\mathcal{T} : \quad x_1 \to x_1 + \pi R_H \ . \tag{2.1}$$

The operator Q_H is the generator of the orbifold that produces the ten-dimensional supersymmetric $E_8 \times E_8$ theory from the ten-dimensional supersymmetric $SO(32)$ theory. Note that if we decompose the representations of the original gauge group $SO(32)$ into those of $SO(16) \times SO(16)$, then Q_H acts with a minus sign on the vector representation and one of the spinor representations of the first $SO(16)$ factor.

This construction yields a supersymmetric nine-dimensional heterotic string model whose partition partition is given by:

$$
\begin{aligned}
Z(\tau) \;=\; & (\operatorname{Im}\tau)^{-7/2} \frac{1}{(\overline{\eta}\eta)^7} \left(\overline{\chi}_V - \overline{\chi}_S \right) \times \Big\{ \mathcal{E}_0(\chi_I^2 + \chi_S^2) + \mathcal{E}_{1/2}(\chi_I \chi_S + \chi_S \chi_I) \\
& + \mathcal{O}_0(\chi_V^2 + \chi_C^2) + \mathcal{O}_{1/2}(\chi_V \chi_C + \chi_C \chi_V) \Big\} \ .
\end{aligned}
\tag{2.2}
$$

Here $\chi_{I,V,S,C}$ are the affine level-one characters of the left-moving $SO(16)$ gauge factors; $\overline{\chi}_{I,V,S,C}$ are the affine level-one characters of the right-moving $SO(8)$ Lorentz group; and the circle-compactification functions $\mathcal{E}, \mathcal{O}$ are defined as

$$
\begin{aligned}
\mathcal{E}_0(\tau, R_H) & \equiv (\overline{\eta}\eta)^{-1} \sum_{\substack{m \in 2\mathbb{Z} \\ n \in \mathbb{Z}}} \exp\left[2\pi i m n \tau_1 \; - \; \pi \tau_2 (m^2 \alpha'/R_H^2 + n^2 R_H^2/\alpha') \right] \\
\mathcal{E}_{1/2}(\tau, R_H) & \equiv (\overline{\eta}\eta)^{-1} \sum_{\substack{m \in 2\mathbb{Z} \\ n \in \mathbb{Z}+1/2}} \exp\left[2\pi i m n \tau_1 \; - \; \pi \tau_2 (m^2 \alpha'/R_H^2 + n^2 R_H^2/\alpha') \right] \\
\mathcal{O}_0(\tau, R_H) & \equiv (\overline{\eta}\eta)^{-1} \sum_{\substack{m \in 2\mathbb{Z}+1 \\ n \in \mathbb{Z}}} \exp\left[2\pi i m n \tau_1 \; - \; \pi \tau_2 (m^2 \alpha'/R_H^2 + n^2 R_H^2/\alpha') \right] \\
\mathcal{O}_{1/2}(\tau, R_H) & \equiv (\overline{\eta}\eta)^{-1} \sum_{\substack{m \in 2\mathbb{Z}+1 \\ n \in \mathbb{Z}+1/2}} \exp\left[2\pi i m n \tau_1 \; - \; \pi \tau_2 (m^2 \alpha'/R_H^2 + n^2 R_H^2/\alpha') \right] \ .
\end{aligned}
\tag{2.3}
$$

Here m and n are respectively the momentum- and winding-mode numbers on the circle, and $\tau_1 \equiv \operatorname{Re}\tau$, $\tau_2 \equiv \operatorname{Im}\tau$.

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These circle-compactification functions have the limits

$$R_H \to \infty : \qquad \mathcal{E}_0, \mathcal{O}_0 \to \left(\frac{R_H}{2\sqrt{\alpha'}}\right)(\sqrt{\tau_2}\,\overline{\eta}\eta)^{-1}; \quad \mathcal{E}_{1/2}, \mathcal{O}_{1/2} \to 0$$

$$R_H \to 0 : \qquad \mathcal{E}_0, \mathcal{E}_{1/2} \to \left(\frac{\sqrt{\alpha'}}{R_H}\right)(\sqrt{\tau_2}\,\overline{\eta}\eta)^{-1}; \quad \mathcal{O}_0, \mathcal{O}_{1/2} \to 0 . \qquad (2.4)$$

Since $(\sqrt{\tau_2}\,\overline{\eta}\eta)^{-1}$ is the partition function of a single uncompactified boson, we see that the relations (2.4) permit us to obtain the partition functions of ten-dimensional string models as the limits of those in nine dimensions (with the divergent radius factors providing the proper overall volume factors). Specifically, using these relations, we see that the partition function Z in Eq. (2.2) has the limits

$$\lim_{R_H \to \infty} Z \sim (\sqrt{\tau_2}\,\overline{\eta}\eta)^{-8}(\overline{\chi}_V - \overline{\chi}_S)(\chi_I^2 + \chi_V^2 + \chi_S^2 + \chi_C^2) = Z_{SO(32)}$$

$$\lim_{R_H \to 0} Z \sim (\sqrt{\tau_2}\,\overline{\eta}\eta)^{-8}(\overline{\chi}_V - \overline{\chi}_S)(\chi_I + \chi_S)^2 = Z_{E_8 \times E_8} . \qquad (2.5)$$

This demonstrates that our nine-dimensional model smoothly interpolates between the ten-dimensional $SO(32)$ and $E_8 \times E_8$ heterotic theories. For generic radius $0 < R_H < \infty$, the massless states of this model consist of spacetime vectors and spinors transforming in the adjoint representation of $SO(16) \times SO(16)$. As $R_H \to \infty$, the $SO(32)$ gauge symmetry is restored through the appearance of extra massless vectors and spinors in the $(\mathbf{16}, \mathbf{16})$ representation of $SO(16) \times SO(16)$, and as $R_H \to 0$ the $E_8 \times E_8$ gauge symmetry is obtained through the appearance of extra vectors and spinors in the $(\mathbf{128}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{128})$ representation. There are no extra massless particles at any finite non-zero radius.

3 The Dual Interpolating Model

Let us now construct the strong-coupling dual for our heterotic interpolating model.

Since the $SO(32)$ heterotic string is conjectured to be dual to the $SO(32)$ Type I string, our dual interpolating model should smoothly connect to the $SO(32)$ Type I string. This suggests that our dual theory should be a smooth interpolation away from the $SO(32)$ Type I theory, and should include this theory at one endpoint. In order to study interpolations of Type I theories, we shall first consider interpolations of Type II theories, for we know that Type I theories can be obtained from Type II theories as orientifolds. In particular, since the $SO(32)$ Type I theory can be realized as the orientifold of the Type IIB theory, we shall begin by considering the nine-dimensional Type II interpolation that connects to the ten-dimensional Type IIB theory at one endpoint.

It turns out that there indeed exists such a Type II model in nine dimensions which connects to the Type IIB string at one endpoint and which is supersymmetric for all radii. This model was explicitly described in Ref. [7], and is similar to one of the models considered in Ref. [9]. This model can be realized by compactifying the Type IIB theory on a circle of radius R_I, and then orbifolding by the action $\mathcal{T}$ where $\mathcal{T}$ is the half-rotation operator given in Eq. (2.1). The partition function of the resulting model is

$$ Z(\tau) \;=\; (\operatorname{Im}\tau)^{-7/2}\,\frac{1}{(\overline{\eta}\eta)^7}\,(\mathcal{E}_0 + \mathcal{E}_{1/2})\,(\overline{\chi}_V - \overline{\chi}_S)\,(\chi_V - \chi_S) \tag{3.1} $$

where we now use the level-one characters of $SO(8)$ for both the left- and right-movers.

This model connects the supersymmetric Type IIB theory at infinite radius with the same model at zero radius. However, as explained in Refs. [9, 10], the Type IIB theory at zero radius is equivalent to the Type IIA theory at infinite radius, and it is therefore the Type IIA theory that should be regarded as the effective ten-dimensional theory that is produced in the $R_I \to 0$ limit. Thus, our supersymmetric nine-dimensional Type II model interpolates between the ten-dimensional Type IIB and Type IIA theories.

It is worth emphasizing that this is not the only nine-dimensional Type II model that interpolates between the Type IIB and Type IIA theories. Another much more trivial example would be the straightforward circle-compactification of the Type IIB theory, *without* the orbifolding by $\mathcal{T}$. As the radius of this circle goes to infinity, such a model would reproduce the Type IIB string; it would also reproduce the Type IIB string at zero radius, which is equivalent to the uncompactified Type IIA string. However, such a trivial compactification would not be suitable for our purposes: upon orientifolding, the resulting theory would not connect theories with different gauge groups, and moreover the soliton of the resulting theory would not have the desired properties. Thus, as we shall see, the half-rotation orbifold element $\mathcal{T}$ is crucial not just on the heterotic side, but also on the dual side.

Given this Type II interpolating model, we can now construct its corresponding open-string orientifold. We take our orientifold generator to be Ω, the worldsheet parity operator. Thus, putting our entire construction together, we see that our open-string interpolating model will be a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold of the supersymmetric Type IIB theory compactified on a circle of radius R_I. The first $\mathbb{Z}_2$ factor corresponds to the worldsheet parity operator Ω, and the second to the orbifold operator $\mathcal{T}$. Because these two operators commute, we will be able to consider the resulting open-string interpolating model either as an orientifold of the Type II interpolation, or as an interpolating orbifold of the $SO(32)$ Type I theory. Of course, as $R_I \to \infty$,

this open-string model will smoothly reproduce the supersymmetric $SO(32)$ Type I theory.

In order to construct this open-string theory, we follow the standard orientifolding procedure [11, 10]. Our conventions and notation will be the same as those of Ref. [7]. We must evaluate eight traces, two each (corresponding to the NS-NS and Ramond-Ramond sectors) for the torus, Klein bottle, cylinder, and Möbius strip. Because this model is supersymmetric, the NS-NS and Ramond-Ramond traces are equal to each other in all cases and provide cancelling contributions. Our traces are then as follows. The total torus trace is simply half of the Type II partition function given in Eq. (3.1). Similarly, defining $q \equiv e^{-2\pi t}$, $f_1(q) \equiv \eta(q^2)$, and $f_i(q) \equiv \sqrt{\vartheta_i(q^2)/\eta(q^2)}$ for $i = 2, 3, 4$, we find that the NS-NS or Ramond-Ramond components of the remaining traces are given as:

$$
\begin{aligned}
K'(t) &= \frac{1}{8} \frac{f_4^8(q)}{f_1^8(q)} \left\{ \sum_{m=-\infty}^{\infty} \left[1 + (-1)^m \right] q^{m^2 a^2/2} \right\} \\
C'(t) &= \frac{1}{8} \frac{f_4^8(q^{1/2})}{f_1^8(q^{1/2})} \left\{ (\operatorname{Tr} \gamma_I)^2 \sum_{m=-\infty}^{\infty} q^{m^2 a^2} + (\operatorname{Tr} \gamma_{\mathcal{T}})^2 \sum_{m=-\infty}^{\infty} (-1)^m q^{m^2 a^2} \right\} \\
M'(t) &= -\frac{1}{8} \frac{f_2^8(q) f_4^8(q)}{f_1^8(q) f_3^8(q)} \times \\
&\qquad \left\{ (\operatorname{Tr} \gamma_\Omega^T \gamma_\Omega^{-1}) \sum_{m=-\infty}^{\infty} q^{m^2 a^2} + (\operatorname{Tr} \gamma_{\Omega\mathcal{T}}^T \gamma_{\Omega\mathcal{T}}^{-1}) \sum_{m=-\infty}^{\infty} (-1)^m q^{m^2 a^2} \right\} . \quad (3.2)
\end{aligned}
$$

Here we have defined $a \equiv \sqrt{\alpha'}/R_I$; the primes on $K'(t)$, $C'(t)$, and $M'(t)$ indicate that these traces do not include the integrations over non-compact momenta; and γ_I, $\gamma_{\mathcal{T}}$, and $\gamma_{\Omega\mathcal{T}}$ respectively indicate the actions of the identity, the orbifold element $\mathcal{T}$, and the orientifold element $\Omega\mathcal{T}$ on the Chan-Paton factors (or equivalently on the nine-branes in the open-string theory).

In order to solve for these γ matrices, we must impose the tadpole anomaly cancellation constraints. In general, tadpole anomalies appear as divergences in the one-loop amplitudes from the $t \to 0$ region of integration of these traces, and are interpreted as arising from the exchange of massless (and possibly tachyonic) string states in the tree channel. In order to extract these divergences, it is simplest to recast the above traces from the loop variable t to the tree variable ℓ, defined as

$$
\ell \equiv \begin{cases} 1/(4t) & \text{Klein bottle} \\ 1/(2t) & \text{cylinder} \\ 1/(8t) & \text{Möbius strip} . \end{cases} \quad (3.3)
$$

The procedure for doing this is standard, and we find that our above traces can be

rewritten as

$$
K'(\ell) = \frac{\ell^{-7/2}}{64\,a}\,\frac{f_2^8(\tilde{q})}{f_1^8(\tilde{q})}\,\sum_{m=-\infty}^{\infty}\left(\tilde{q}^{2m^2/a^2} + \tilde{q}^{2(m+1/2)^2/a^2}\right)
$$

$$
C'(\ell) = \frac{\ell^{-7/2}}{128\,a}\,\frac{f_2^8(\tilde{q})}{f_1^8(\tilde{q})}\left\{(\mathrm{Tr}\,\gamma_I)^2\sum_{m=-\infty}^{\infty}\tilde{q}^{m^2/2a^2} + (\mathrm{Tr}\,\gamma_{\mathcal{T}})^2\sum_{m=-\infty}^{\infty}\tilde{q}^{(m+1/2)^2/2a^2}\right\}
$$

$$
M'(\ell) = -\frac{\ell^{-7/2}}{1024\,a}\,\frac{f_2^8(\tilde{q}^2)\,f_4^8(\tilde{q}^2)}{f_1^8(\tilde{q}^2)\,f_3^8(\tilde{q}^2)}\times
$$
$$
\left\{(\mathrm{Tr}\,\gamma_\Omega^T\gamma_\Omega^{-1})\sum_{m=-\infty}^{\infty}\tilde{q}^{2m^2/a^2} + (\mathrm{Tr}\,\gamma_{\Omega\mathcal{T}}^T\gamma_{\Omega\mathcal{T}}^{-1})\sum_{m=-\infty}^{\infty}\tilde{q}^{2(m+1/2)^2/a^2}\right\} \quad (3.4)
$$

where $\tilde{q} \equiv e^{-2\pi\ell}$. Taking the $\ell \to \infty$ limit, and recalling that the nine-dimensional Klein-bottle, cylinder, and Möbius-strip one-loop amplitudes $\int_0^\infty d\ell\,\ell^{7/2}\,\{K', C', T'\}$ have relative prefactors of 512, 1, and 512 respectively, we see that the NS-NS and Ramond-Ramond tadpoles will both be cancelled for $R_I > 0$ if

$$
64 + \frac{1}{16}\,(\mathrm{Tr}\,\gamma_I)^2 - 4\,(\mathrm{Tr}\,\gamma_\Omega^T\gamma_\Omega^{-1}) = 0 \,. \quad (3.5)
$$

These tadpoles cancel for any non-zero radius by choosing γ_Ω symmetric and γ_I to be 32-dimensional identity matrix. Although we do not obtain a restriction on $\gamma_{\mathcal{T}}$, we can choose

$$
\gamma_{\mathcal{T}} = \begin{pmatrix} \mathbf{1}_{16} & 0 \\ 0 & -\mathbf{1}_{16} \end{pmatrix}\,, \quad (3.6)
$$

corresponding to the gauge group $SO(16)\times SO(16)$. With this choice, we obtain open-string states consisting of a nine-dimensional vector supermultiplet in the adjoint of $SO(16) \times SO(16)$. When combined with the gravitational and Kaluza-Klein states from the closed-string sector, this agrees exactly with the spectrum of massless states of the heterotic interpolating model of Sect. 2.

Let us now discuss the $R_I \to 0$ limit. In this case, we must be more careful when extracting the tadpole divergences. In the $R_I \to 0$ limit, the momentum sums in Eq. (3.4) can be evaluated exactly:

$$
\lim_{a\to\infty}\sum_{m=-\infty}^{\infty}\tilde{q}^{Am^2/a^2} = \lim_{a\to\infty}\sum_{m=-\infty}^{\infty}\tilde{q}^{A(m+1/2)^2/a^2} = \frac{a}{\sqrt{2\ell A}} \quad (3.7)
$$

for any normalization factor A. Thus, as $\ell \to \infty$, the leading behavior of these traces is

$$
\begin{aligned}
K'(\ell) &\sim (1/4)\,\ell^{-4}\,[1+...] \\
C'(\ell) &\sim (1/8)\,\ell^{-4}\,\{(\mathrm{Tr}\,\gamma_I)^2+(\mathrm{Tr}\,\gamma_{\mathcal{T}})^2\}\,[1+...] \\
M'(\ell) &\sim -(1/128)\,\ell^{-4}\,\{(\mathrm{Tr}\,\gamma_\Omega^T\gamma_\Omega^{-1})+(\mathrm{Tr}\,\gamma_{\Omega\mathcal{T}}^T\gamma_{\Omega\mathcal{T}}^{-1})\}\,[1+...] \,. \quad (3.8)
\end{aligned}
$$

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The change in the power of ℓ reflects the change in the effective dimensionality of the theory. For such a ten-dimensional theory, the one-loop amplitudes now have the form $\int_0^\infty d\ell\, \ell^4 \{K', C', M'\}$ with relative prefactors 1024, 1, and 1024 respectively. We thus obtain the tadpole cancellation constraint

$$256 \;+\; \frac{1}{8}\left[(\mathrm{Tr}\,\gamma_I)^2 + (\mathrm{Tr}\,\gamma_{\mathcal{T}})^2\right] \;-\; 8\left[(\mathrm{Tr}\,\gamma_\Omega^T\gamma_\Omega^{-1}) + (\mathrm{Tr}\,\gamma_{\Omega\mathcal{T}}^T\gamma_{\Omega\mathcal{T}}^{-1})\right] \;=\; 0\;. \qquad (3.9)$$

The only solution to this equation is one for which $\gamma_{\mathcal{T}}$ is taken to be the 32-dimensional identity matrix. Unfortunately, the continuity of our nine-dimensional Type I interpolating model prevents us from making this new choice for $\gamma_{\mathcal{T}}$; we must continue to have $\gamma_{\mathcal{T}}$ as given in Eq. (3.6). However, given this choice, there is no solution to the constraint equation (3.9). Specifically, this equation cannot be satisfied by adding or subtracting nine-branes (*i.e.*, changing the rank of the gauge group), or even by adding anti-branes. This is, of course, to be expected, for we know from ten-dimensional anomaly cancellation arguments that there is no consistent supersymmetric theory in ten dimensions with gauge group $SO(16) \times SO(16)$.

4   The $E_8 \times E_8$ Soliton

The resolution to this "puzzle" is that there are extra non-perturbative states from the D1-brane soliton of this theory that become massless as $R_I \to 0$. Let us therefore now discuss the soliton of this theory. Our procedure for analyzing this soliton will be similar to the procedure followed in Ref. [7], and consequently we shall summarize here only the salient features.

Our open-string model contains a solitonic object or D1-brane. Solitons of the $SO(32)$ Type I theory corresponding to $SO(32)$ heterotic strings were found as classical solutions in Ref. [12], and were constructed from a collective-coordinate expansion on the D1-brane in Ref. [2]. In this section, we will construct the soliton of our Type I interpolating model by compactifying the $SO(32)$ soliton on a circle and projecting its worldsheet fields by the orbifold element $\mathcal{Y} \equiv \mathcal{T}\gamma_{\mathcal{T}}$.

Note that the tension of the soliton is $T_S \sim T_F/\lambda_I^{(10)}$ where $T_F = 1/2\pi\alpha'$ is the fundamental string tension and where $\lambda_I^{(10)}$ is the ten-dimensional Type I coupling. If we were to choose the soliton to lie along any direction orthogonal to the compact direction, the soliton would be very heavy for all perturbative values of the coupling and would not behave as a fundamental object. Thus, we shall choose the soliton to lie along the compact x_1 direction because this choice allows states of the soliton to become nearly massless at perturbative values of the couplings for sufficiently small R_I.

In quantizing the fields on the soliton, we find that the quantized zero-mode momentum of Type I bosons in the x_1 direction becomes the quantized oscillator moding of the worldsheet fields on the soliton. Moreover, the Type I theory provides a restriction on the left- and right-moving momenta of the form $p_1^L = p_1^R$, and this restriction becomes the worldsheet restriction $L_0 = \overline{L_0}$ on the left- and right-moving Virasoro generators on the soliton.

Before the projection, the fields on the $SO(32)$ soliton are as follows. The right-movers along x_1 consist of eight transverse spatial bosonic fields X^i, $i = 1, ..., 8$, and a Green-Schwarz fermion S^a, $a = 1, ..., 8$, of definite chirality. The left-movers consist of eight transverse spatial bosonic fields X^i and 32 worldsheet fermions ψ^A, $A = 1, ..., 32$. There is also a GSO projection acting on the left-moving fermions.

After we compactify this soliton on a circle, we will project by the $\mathbb{Z}_2$ element $\mathcal{Y} = \mathcal{T}\gamma_{\mathcal{T}}$. Recall that $\gamma_{\mathcal{T}}$ is the Type I Wilson line that breaks $SO(32)$ to $SO(16) \times SO(16)$. It turns out that the action of $\gamma_{\mathcal{T}}$ on the worldsheet fields of the $SO(32)$ soliton exactly corresponds to the orbifold operator Q_H that we discussed in Sect. 2. Since $\gamma_{\mathcal{T}}$ does not act on the Type I spacetime, the spacetime action of $\mathcal{Y}$ is solely due to $\mathcal{T}$ itself. This effectively halves the radius of the circle of compactification. All worldsheet bosons of the soliton will thus be restricted to integral moding such that these fields are periodic on the half-radius circle.

After the projection, there will be a variety of resulting sectors. One group of sectors will be those for which all fields have the usual modings on the half-radius circle (such that the fields are periodic on the half-radius circle). However, the reduction to the half-radius circle implies that there will also be additional sectors for which some fields may be anti-periodic on the half-radius circle (provided they are consistent with the symmetries preserved by the orbifold). Such unexpected modings on the half-radius circle arise naturally when the original fields on the circle of radius R_I are decomposed into fields on the circle of half-radius $R_I/2$. Taken together, then, all of these sectors will describe the Ramond sector of the resulting soliton, and just as for the supersymmetric $SO(32)$ soliton, we will assume that a corresponding Neveu-Schwarz sector arises non-perturbatively.

Because of the unbroken supersymmetry in our theory, we expect that the massless states of our soliton will also be massless at strong coupling. Note that the soliton preserves half the supersymmetry of the orientifold, resulting in a BPS condition for the soliton. We will therefore analyze the soliton at weak coupling. However, unlike the case of the non-supersymmetric solitons discussed in Ref. [7], we will project with $\mathcal{Y}$ separately on left- and right-movers, and in the end we will allow combinations of left- and right-moving sectors with different types of modings.

Performing the projection by $\mathcal{Y}$ is straightforward, and we can summarize the

results in the following table:

right-movers		left-movers			
		Ramond		NS	
$\mathcal{Y}$	sector	$\mathcal{Y}$	sector	$\mathcal{Y}$	sector
$+1$	$\overline{V_8}$	$+1$	$S_{16}^{(1)}S_{16}^{(2)}$	$+1$	$I_{16}^{(1)}I_{16}^{(2)}$
$+1$	$\overline{S_8}$	-1	$C_{16}^{(1)}C_{16}^{(2)}$	-1	$V_{16}^{(1)}V_{16}^{(2)}$
		$+1$	$I_{16}^{(1)}V_{16}^{(2)}$	$+1$	$S_{16}^{(1)}C_{16}^{(2)}$
		-1	$V_{16}^{(1)}I_{16}^{(2)}$	-1	$C_{16}^{(1)}S_{16}^{(2)}$
		$+1$	$I_{16}^{(1)}S_{16}^{(2)}$	$+1$	$S_{16}^{(1)}I_{16}^{(2)}$
		$+1$	$V_{16}^{(1)}C_{16}^{(2)}$	$+1$	$C_{16}^{(1)}V_{16}^{(2)}$
		$+1$	$S_{16}^{(1)}V_{16}^{(2)}$	-1	$V_{16}^{(1)}S_{16}^{(2)}$
		$+1$	$C_{16}^{(1)}I_{16}^{(2)}$	-1	$I_{16}^{(1)}C_{16}^{(2)}$

$$(4.1)$$

In this table, we have indicated the resulting right- and left-moving sectors by specifying their representations under the right-moving transverse Lorentz group $SO(8)$ and the two left-moving $SO(16)$ gauge factors respectively. We also indicated their corresponding eigenvalues under $\mathcal{Y}$.

As our last step, in order to construct our final soliton theory, we must join these left- and right-moving sectors together. Our procedure for doing this is as follows. First, as discussed above, we impose the requirement that $L_0 = \overline{L_0}$. Second, we require that the exchange symmetry of the two $SO(16)$ gauge factors be preserved. Third, we require that all left/right combinations be invariant under $\mathcal{Y}$.

Remarkably, imposing these three restrictions simultaneously, we find that we obtain precisely the eight sectors that comprise the $E_8 \times E_8$ heterotic string:

$$\overline{V_8}\,I_{16}^{(1)}I_{16}^{(2)} \;,\;\; \overline{V_8}\,S_{16}^{(1)}S_{16}^{(2)} \;,\;\; \overline{V_8}\,I_{16}^{(1)}S_{16}^{(2)} \;,\;\; \overline{V_8}\,S_{16}^{(1)}I_{16}^{(2)}$$
$$\overline{S_8}\,I_{16}^{(1)}I_{16}^{(2)} \;,\;\; \overline{S_8}\,S_{16}^{(1)}S_{16}^{(2)} \;,\;\; \overline{S_8}\,I_{16}^{(1)}S_{16}^{(2)} \;,\;\; \overline{S_8}\,S_{16}^{(1)}I_{16}^{(2)} \;. \qquad (4.2)$$

Thus, we see that the $E_8 \times E_8$ heterotic string can indeed be realized as the D-string soliton of our Type I interpolating model. Moreover, we see from this analysis that at $R_I = 0$, the soliton provides massless spacetime vectors and spinors in the $(\mathbf{128}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{128})$ representation of $SO(16) \times SO(16)$. These additional states thus enhance the gauge group of our orientifold theory to $E_8 \times E_8$, and thereby cancel the anomaly that we found earlier.

5   Analysis and Discussion

Let us now analyze our dual theory as a function of its radii and couplings. This will be necessary in order for us to properly interpret our results. Specifically, we seek

to know the conditions under which our $E_8 \times E_8$ soliton indeed becomes massless at $R_I = 0$.

We begin by recalling some basic relations. If we compactify M-theory first on a line segment of radius (length) $R_H^{(11)}$ and then on a circle of radius $R_H^{(10)}$, the resulting theory is equivalent to the ten-dimensional $E_8 \times E_8$ heterotic string with coupling λ_H compactified on a circle of radius R_H', where

$$R_H^{(11)} = \lambda_H^{2/3} , \qquad R_H^{(10)} = \frac{R_H'}{\lambda_H^{1/3}} . \tag{5.1}$$

Here $R_H^{(10,11)}$ are given in M-theory units, while R_H' is given in the usual string units. Note that the prime on R_H' is present in order to distinguish this radius variable from R_H, the radius of our interpolating heterotic model in Sect. 2. Specifically, our interpolating model at $R_H = 0$ is T-dual to the present theory at $R_H' = \infty$.

The strong-coupling dual of this compactification of M-theory can be realized by exchanging the order of the compactifications: first we compactify M-theory on a circle of radius $R_{I'}^{(11)}$ and then on a line segment of radius (length) $R_{I'}^{(10)}$. This produces the Type I$'$ theory compactified on a circle of radius $R_{I'}$ with coupling $\lambda_{I'}$, where

$$R_{I'}^{(11)} = \lambda_{I'}^{2/3} , \qquad R_{I'}^{(10)} = \frac{R_{I'}}{\lambda_{I'}^{1/3}} . \tag{5.2}$$

Once again, $R_I^{(10,11)}$ are given in M-theory units, while R_I is given in the usual string units. Of course, these two theories are strong-coupling duals of each other only if $R_H^{(11)} = R_{I'}^{(10)}$ and $R_H^{(10)} = R_{I'}^{(11)}$. Finally, in order to realize the Type I theory, we can T-dualize the Type I$'$ theory. This produces the Type I theory formulated at radius R_I and coupling λ_I, where

$$R_I = \frac{1}{R_{I'}} , \qquad \lambda_I = \frac{\lambda_{I'}}{R_{I'}} . \tag{5.3}$$

Let us now consider the mass of the soliton. In general, the soliton has mass $M_{\rm sol} = T_F R_I / \lambda_I$ where $T_F = (2\pi\alpha')^{-1}$ is the fundamental string tension. Using the above relations, we can recast this expression in terms of the fundamental heterotic variables (R_H', λ_H), obtaining

$$M_{\rm sol} = \frac{\lambda_H^{1/2}}{(R_H')^{3/2}} . \tag{5.4}$$

Moreover, if our results are to be consistent, we must have $M_{\rm sol} \to 0$ *at the same time* that $R_I \to 0$. Since

$$R_I = \frac{1}{(R_H')^{1/2} \lambda_H^{1/2}} , \tag{5.5}$$

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we see that these two equations do not have simultaneous solutions unless $R'_H \to \infty$. This is equivalent to $R_H \to 0$ for our interpolating model, which agrees with our expectations on the heterotic side. We are then permitted to have $\lambda_H \sim (R'_H)^\alpha$ where $-1 < \alpha < 3$. Thus, although $R'_H \to \infty$, we see that λ_H is not constrained. Note that these constraints imply that the Type I' coupling $\lambda_{I'} \to \infty$.

Finally, let us now consider how our results compare with those of Ref. [8]. In Ref. [8], the problem of realizing the $E_8 \times E_8$ gauge symmetry was considered from the Type I' point of view. In such a picture, the problem reduces to showing the existence of suitable bound states between mixtures of zero-branes and eight-branes. In order to investigate the existence of such bound states, the authors of Ref. [8] make a Born-Oppenheimer approximation which neglects the non-abelian degrees of freedom involved in short-distance physics. Note that the existence of certain D0-bound states has been argued from a different perspective in Ref. [13]. In this paper, by contrast, we have taken a Type I approach and have found a D1-brane soliton with exactly the worldsheet fields of the $E_8 \times E_8$ heterotic string. This soliton satisfies a BPS condition, so that its tension is not renormalized [14]. Moreover, in the strong-coupling or small-radius limit, our soliton can be expected to behave fundamentally as an $E_8 \times E_8$ string, and we know that this theory is a stable solution to the string equations of motion. Thus we have in some sense proven the existence of all of the states necessary for correspondence with M-theory from the Type I point of view.

It is noteworthy that, just as for the heterotic model, there is exactly one Type I model that interpolates from $SO(32)$ gauge symmetry at infinite radius to $E_8 \times E_8$ gauge symmetry at zero radius. Indeed, it is exactly this interpolation — achieved through the use of the half-rotation operator $\mathcal{T}$ — that enables us to realize the $E_8 \times E_8$ theory on the soliton. We also see that at $R_I = 0$, the M-theory description is essential since the Type I' theory is infinitely coupled and ten-dimensional Lorentz invariance is restored. Thus, our Type I interpolating model succeeds in continuously connecting the Type I $SO(32)$ theory to M-theory compactified on a line segment.

Acknowledgments

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